Lanchester Laws: Model of Common War

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$$\begin{cases} -\alpha, -\delta = \text{operational losses} \propto \text{social capital (cohesion)}, \\ -\beta, -\gamma = \text{battlefield losses} \propto \text{scope (battle wisdom)}, \\ \eta cos(t), \zeta cos(t) = \text{reserves} \propto \text{material capital (supplies)}, \\ \\ a = -\alpha; b = -\beta; c = -\gamma; d = -\delta; f = \eta; h = \zeta, \\ \\ \vec{X}'(t) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{X}(t) + \begin{bmatrix} fcos(t) \\ hcos(t) \end{bmatrix}, \\ \\ \vec{X}(0) = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \\ \\ g = \frac{a+d}{2}, \\ \\ \lambda = \frac{\sqrt{a^2+2ad+4cb+d^2}}{2}. \end{cases}.$$

$$\vec{X}(t) = \vec{X}_h(t) + \vec{X}_p(t) = \vec{X}_h(t) + \begin{bmatrix} x_p(t) \\ y_p(t) \end{bmatrix}.$$

$$\vec{X}_p(0) = \begin{bmatrix} x_p(0) \\ y_p(0) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{g+\lambda-d}{2\lambda}[\frac{-f}{g+\lambda} - \frac{hb}{(g+\lambda-d)(g+\lambda)}] \\ -\frac{g-\lambda-d}{2\lambda}[\frac{f}{-g+\lambda} + \frac{hb}{(g-\lambda-d)(-g+\lambda)}] \\ \\ \frac{(g-\lambda-d)(g+\lambda-d)}{2\lambda b}\{[\frac{f}{g+\lambda} + \frac{hb}{(g+\lambda-d)(g+\lambda)}] \\ -[\frac{f}{-g+\lambda} + \frac{hb}{(g-\lambda-d)(-g+\lambda)}]\} \end{bmatrix}.$$

$\vec{X}(t) =$

$$\frac{1}{2\lambda b}\Big[$$
$$\Big\{[g+\lambda-d][(b)(y_0 - y_p(0)) + (x_0 - x_p(0))(g-\lambda-d)]\Big\}\ \boldsymbol{e^{(g-\lambda)t}} \begin{bmatrix} \boldsymbol{\frac{-b}{g+\lambda-d}} \\ \mathbf{1} \end{bmatrix}$$
$$+ \Big\{[(x_0 - x_p(0))(g-\lambda-d)(2\lambda)] + [g+\lambda-d][(b)(y_0 - y_p(0)) + (x_0 - x_p(0))(g-\lambda-d)]\Big\}\ \boldsymbol{e^{(g+\lambda)t}} \begin{bmatrix} \boldsymbol{\frac{-b}{g-\lambda-d}} \\ \mathbf{1} \end{bmatrix}$$
$$\Big]$$
$$+ \begin{bmatrix} \{\frac{1}{2\lambda}\}\{[g+\lambda-d][(sin(t))(f + \frac{hb}{g+\lambda-d}) - (cos(t))(\frac{f}{g+\lambda} + \frac{hb}{(g+\lambda-d)(g+\lambda)})] \\ -[g-\lambda-d][(sin(t))(f + \frac{hb}{g-\lambda-d}) + (cos(t))(\frac{f}{-g+\lambda} + \frac{hb}{(g-\lambda-d)(-g+\lambda)})]\} \\ \\ \{\frac{(g-\lambda-d)(g+\lambda-d)}{2\lambda b}\}\{[(sin(t))(f + \frac{hb}{g-\lambda-d}) + (cos(t))(\frac{f}{-g+\lambda} + \frac{hb}{(g-\lambda-d)(-g+\lambda)})] \\ -[(sin(t))(f - \frac{hb}{g+\lambda-d}) - (cos(t))(\frac{f}{g+\lambda} + \frac{hb}{(g+\lambda-d)(g+\lambda)})]\} \end{bmatrix}.$$